PHYSICAL REVIEW E **71**, 051708 (2005)

Surface anchoring and dynamics of jump-wise director reorientations in planar cholesteric layers

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(Received 24 August 2004; published 9 May 2005)

A theoretical investigation is made into the dynamics of pitch jumps in cholesteric liquid-crystal layers having finite strength surface-anchoring conditions. A presentation is given of general formulations which connect the dynamics of pitch jumps with the key material parameters such as the viscosity, the specific form of the anchoring potential, and the dimensionless parameter $S_d = K_{22}/Wd$, where K_{22} is the elastic modulus, W is the depth of the anchoring potential, and d is the layer thickness. To illustrate the dependence of the pitch jump dynamics upon the shape and strength of the anchoring potential, we investigate two sets of different model surface-anchoring potentials for a jump mechanism that is connected with the slipping of the director at a surface over the barrier of the anchoring potential. Two types of "narrow" well potentials that are natural extensions of the more familiar "wide" potentials are considered: one type is based upon the well-known Rapini-Papoular potential and the other upon the *B* potential, introduced in Belyakov, Stewart, and Osipov, JETP **99**, 73 (2004). Calculations for the unwinding (winding) of the helix in the process of the jump were performed to investigate the case of infinitely strong anchoring on one surface and finite anchoring on the other, which is important in applications. The results show that an experimental investigation of the dynamics of the pitch jumps will allow one to distinguish different shapes of the finite strength anchoring potential, and will, in particular, provide a means for determining whether or not the well-known Rapini-Papoular anchoring potential is the best suited potential relevant to the dynamics of pitch jumps in cholesteric layers with finite surface-anchoring strength.

DOI: 10.1103/PhysRevE.71.051708 PACS number(s): 61.30.Dk, 61.30.Hn

I. INTRODUCTION

The influence of finite surface anchoring and thermodynamic fluctuations upon the temperature dependence of pitch variations in planar layers of cholesteric liquid crystal have recently revealed [1,2] some interesting effects that are of importance not only in the general physics of liquid crystals but also in their practical applications. As has been known for some time, the temperature evolution of cholesteric liquid-crystal structures in samples with finite anchoring energy may be continuous over some range of temperature with jumplike changes at definite temperature points [3–5] and strong hysteresis when the temperature is reversed [5,6]. Some of these problems have been investigated recently both theoretically and experimentally [1,2,6,7]. It has also been shown that in some cases such systems may possess several metastable states [8,9]. The dynamics of thin cholesteric layers may be even more important from the application point of view because of their potential use in the construction of multistable displays and switches. From a more general point of view, such systems belong to a wider class of liquid-crystal systems in a strongly restricted geometry with properties being controlled to a large extent by surface anchoring [4,10,11]. The general dynamics of thin cholesteric, or even nematic, layers with special surface conditions and possibly multiple metastable states is a very difficult computational problem. Thus only a few particular cases have been investigated so far. This includes, for example, the theoretical description [12] of the switching between two bistable states [13,14] which was experimentally observed by Barberi and Durand [13]. Another example is harmonic oscillation of the helical structure in a cholesteric layer subject to a pulsed electric field perpendicular to the helical axis [15]. This type of experiment may be used to measure the rotational viscosity γ_1 [16].

Very recently, theoretical modeling of the jump dynamics has received new impetus because it has been found that pitch jumps have revealed themselves in the lasing frequency jumps that occur in mirrorless temperature-controlled lasing in chiral liquid crystals [17,18]. One notes that in real systems, pitch jumps often do not occur simultaneously in the whole cell. Instead a domain wall (which in the case of a straight wall moves with a certain stationary velocity) is formed which separates regions that have different values of the pitch. Such domain walls (which have very recently been observed by Kuczynski [6] and Coles and co-workers [18]) interfere with lasing, and it is important to find ways to control them. It is important to note here that the stationary velocity of the domain wall is determined by that very relaxation process of the helical structure in a thin layer which is investigated in this article. Other systems where dynamics of jump-wise variations of the pitch may also be important are cholesteric layers with large flexoelectric coefficients and weak surface anchoring subject to external electric fields [19]. In such systems, one finds the so-called longitudinal flexoelectric domains [20–22]. Such domains also exist in nematics (where they are also known as "variable grating mode" [[23], pp. 105–108] because the period of the domains is inversely proportional to the applied voltage). In cholesteric cells, however, the orientation of the domains depends on the number of half-turns of the helix between the cell boundaries. This number is changed after the pitch jump, and thus the aforementioned relaxation of the helical structure after the jump should control the reorientation of such domains in an oscillating field.

In this present article, which is a continuation of our preliminary study [7], we investigate pitch jump dynamics in a relatively thin planar cholesteric layer for the simplest case of an infinitely strong anchoring on one of the surfaces and weak anchoring on the other. This case corresponds to large values of the dimensionless parameter $S_d = K_{22}/Wd$, where K_{22} is the twist elastic constant, W is the depth of the anchoring potential, and d is the layer thickness. S_d is the nondimensionalized (with respect to d) extrapolation length K_{22}/W. As discussed in previous articles, in the case of weak anchoring and relatively thin layers the jump-wise changes of the pitch may occur without formation of defects because it is always more energetically favorable to adjust the director orientation at the surface than to form a defect core. The same conclusion is also valid for dynamics of the helix in the course of a jump. In the case $S_d \sim 1$, the dynamical torques are of the same order as the equilibrium ones, and they are not sufficiently strong to cause any disruption of the director distribution, including any deviation from the *xy* plane. It should be noted that even in this simple case, the relaxation of the helical structure after the pitch jump is very different from the dynamics of a helix unwinding in a field. It will be shown that for finite values of surface-anchoring, the characteristic relaxation times are much larger than the corresponding relaxation times in the unwinding process (which have been estimated, for example, in [24]). This result enables one to justify the so-called quasistatic approximation, which assumes that the homogeneous helical structure remains undistorted and only the pitch is slowly changing with time. In this case, the relaxation of the helical structure in the cell occurs without any orientational waves traveling from one surface to the other. The quasistatic approximation and its limits of applicability are investigated and derived in detail in the Appendix.

The study of dynamics in the course of a pitch jump in this simple case can be considered as the first step in the modeling of more complex dynamical effects including, in particular, the movement of domain walls in thin cholesteric layers and switching between multistable states, which will be undertaken in the near future. At the same time, even in this simple case, the study reveals some interesting features related to the effects of restricted geometry. In particular, the pitch jump dynamics is essentially dependent on the shape of the surface-anchoring potential, which may be different from the Rapini-Papoular form [7]. There is no doubt that the Rapini-Papoular potential is a reasonable approximation for any surface potential for small deviation angles. This is because the Rapini-Papoular potential is not a complete model but only the first term of the systematic expansion of any potential around the equilibrium anchoring angle which should be valid at least for sufficiently small deviations. This potential is frequently used not only because of its simplicity but also because usually one studies phenomena related to surface anchoring which are determined only from the consideration of small deviation angles, where the shape of the potential is approximately quadratic in the deviation angle. Conventional experimental techniques are able to test the form of the potential for relatively small or medium (if nonlinear effects are taken into consideration) deviation angles. The problem is that jump dynamics is particularly sensitive to the shape of the potential at large deviations, i.e., around its maximum. The corresponding relaxation time is a unique quantity which can yield some qualitative information about the form of the potential *maximum* (and not the minimum where the Rapini-Papoular potential is justified), i.e., about the form of the potential barrier which the director has to overcome after the pitch jump.

The natural way to construct a surface potential for large deviations from the equilibrium could be to add a number of higher-order terms to the Rapini-Papoular potential as indeed has been proposed by several authors including Barberi *et al.* [25] and Yoneya *et al.* [26]. Such expressions, however, make real sense only if the deviation angles are not too large, i.e., if the expansion converges rapidly. For very large deviations, when the angle is about $\pi/2$, it is difficult to decide how many terms one has to keep in the expansion. In addition, the coefficients in these terms are generally unknown, and as a result the model anchoring potential will depend on a large number of independent parameters, which makes it difficult to arrive at any general conclusions. In this paper, we have taken a different approach. Instead of using an expansion of the general potential, we have selected a family of simple model potentials which depend on a single model parameter that characterizes the potential depth. All these potentials, of course, reduce to the Rapini-Papoular potential for small deviation angles. At the same time, different potentials considered in this paper differ dramatically in the vicinity of the maximum of the potential. As discussed below in Sec. III, the family of such potentials mainly includes the qualitatively different limiting cases when the potential barrier is very sharp or very broad, respectively. This approach enables one to study the influence of some crude qualitative features of the anchoring potential on the order of magnitude of the relaxation time of the helical structure in a thin cell. Such relaxation times can be measured experimentally and the results may shed some light on these qualitative shapes of the surface potential barrier. However, it should be noted that such an approach may not be justified for a general study. For a particular physical system with known surface properties, it may be a better idea to model the potential by a few terms taken from the general expansion, especially if different terms may have different physical meaning, i.e., they may be determined by different interactions or surface effects. In this case, one can still use the general formulas obtained in this paper, which are valid for any form of the potential.

The plan of this article is as follows. In Sec. II, we discuss the nature of jump dynamics via the dissipation function and special forms for the director orientation angle. The usual Rapini-Papoular (RP) and B potentials are also summarized. Narrow well potentials RP_n and B_n, which are modifications of the RP and B potentials that are indexed by n, are introduced in Sec. III. An identification of the jump angle of the director as a function of the dimensionless parameter S_d is also made. This leads naturally to a discussion of the jump dynamics for narrow well potentials in Sec. IV, where the temporal behavior of the director angle at the layer surface during a jump is presented in Figs. 7 and 8, with a compari-

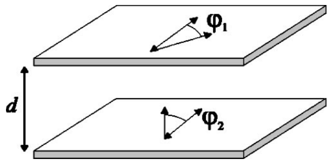

FIG. 1. The case of nonidentical anchoring at the surfaces of a cholesteric layer. The double-headed arrows represent the alignment direction at the surfaces. (For infinitely strong anchoring at the lower surface, $\varphi_2=0$, and for finite anchoring at the upper surface, $\varphi=\varphi_1$.)

son of the results for RP_n and B_n potentials being given in Fig. 9. In Sec. V, the dependence of the switching times upon the normalized anchoring strength and sample thickness have been calculated in Fig. 10; a comparison of the results for the temporal behavior at different values of S_d appears in Fig. 11. The article closes in Sec. VI with some conclusions and a brief discussion.

II. JUMP DYNAMICS

We investigate the dynamics of pitch jumps in a cell of cholesteric liquid crystal with strong anchoring at one surface (at $z=0$) and relatively weak anchoring at the other surface (at $z=d$, where d is the layer thickness), as shown in Fig. 1, where the double-headed arrows indicate the alignment direction at the surfaces. Similar to our previous article [7], we consider the reorientation of the director in the bulk and assume hydrodynamic flow is negligible with the director being everywhere parallel to the plane of the surfaces, i.e., parallel to the xy plane.

The director distribution under the above assumptions is completely specified by the azimuthal angle $\phi(z,t)$. In the absence of flow, the director relaxation is described by the general equation [27–29]

$$\frac{dF}{dt} = -\int_{\Omega} \mathcal{D} d\Omega, \tag{2.1}$$

where F is the total free energy of the liquid-crystal cell, $\mathcal{D}$ is the Rayleigh dissipation function, and Ω is the sample volume. It is further assumed that the director distribution in the layer is quasistatic, that is, the helical structure within the layer remains undistorted and corresponds to some value of the pitch which is changing with time. In this case, the azimuthal angle in the bulk and its derivative can be expressed in terms of the angle $\varphi(t)$ at the surface $z=d$ via the relations

$$\phi(z,t) = \frac{z}{d}\varphi(t) \quad \text{and} \quad \frac{\partial \phi}{\partial t} = \frac{z}{d}\frac{d\varphi}{dt}. \tag{2.2}$$

According to Eq. (2.2), the angle $\phi(z,t)$ vanishes at the surface $z=0$ which is characterized by strong anchoring, and is equal to $\varphi(t)$ at the surface $z=d$. This quasistatic approximation and its limits of validity are derived and considered in detail in the Appendix.

The free energy in Eq. (2.1) is the volume integral of the well known Frank distortion energy of the cholesteric phase

$$F = \tfrac{1}{2}K_{11}(\nabla \cdot \mathbf{n})^2 + \tfrac{1}{2}K_{22}(\mathbf{n}\cdot \nabla \times \mathbf{n} + q)^2 + \tfrac{1}{2}K_{33}(\mathbf{n}\times \nabla \times \mathbf{n})^2. \tag{2.3}$$

In a homogeneous planar cholesteric cell without defects, only the twist deformation is present and the director is expressed as

$$n_x = \cos(qz), \quad n_y = \sin(qz), \quad n_z = 0, \tag{2.4}$$

where q is the wave vector of the helical structure. Substitution of Eq. (2.2) into Eq. (2.1) yields the following simple expression for the total free energy of the cholesteric layer F [1–4] in terms of the single variable φ (which has the meaning of the director orientation angle at the surface with finite anchoring):

$$F(T) = W_s(\varphi) + \frac{K_{22}}{2d}[\varphi - \varphi_0(T)]^2, \tag{2.5}$$

where $W_s(\varphi)$ is the surface anchoring potential at $z=d$ and K_{22} is the twist elastic constant. The angle φ is related to the actual wave vector in the cell q by the simple relation $\varphi=qd$. Here the angle $\varphi_0(T)$ is the external parameter determined by the director rotation angle at $z=d$ in the absence of anchoring, i.e., the quantity dependent on the equilibrium wave vector $q_0(T)$ of the helix in an infinite sample of the cholesteric liquid crystal $[\varphi_0(T)=q_0(T)d]$. In terms of the angle φ, the dissipation function $\mathcal{D}$ [27–29] can then be expressed as

$$\mathcal{D} \equiv \gamma_1\left(\frac{\partial \phi}{\partial t}\right)^2 = \gamma_1\left(\frac{z}{d}\right)^2\left(\frac{d\varphi}{dt}\right)^2, \tag{2.6}$$

where γ_1 is the rotational viscosity.

The equation determining the equilibrium value of φ as a function of the temperature [or $\varphi_0(T)$] is obtained from Eq. (2.5) and is given by

$$\frac{dW_s}{d\varphi} + \frac{K_{22}}{d}[\varphi - \varphi_0(T)] = 0. \tag{2.7}$$

Here, and below, the angles φ and $\varphi_0(T)$ denote the deviation of the director at the surface with finite anchoring from the alignment direction (this involves an inessential change of the origin when assessing the director orientation angle). An analysis of Eq. (2.7) [1,2,7] shows that a smooth change in the director deviation angle φ is possible while φ is less than some critical angle φ_c. Upon φ achieving the critical value φ_c, a jumplike change of the pitch occurs. For $S_d > 1/2\pi$, the transition to the unique new configuration of the helix occurs that differs by one in the number of half-turns N. In this case, it is possible to restrict the range of values of φ to the interval $[-\pi/2,\pi/2]$ using the formula $\varphi=N\pi+\varphi'$, where the integer $N=\mathrm{int}[\varphi/\pi]$ is the number of half-turns within the layer thickness. All solutions for φ' fit into the domain $[-\pi/2,\pi/2]$. For the remainder of this article, we only use the variable φ', with the prime dropped for simplicity. The critical value of the director deviation angle φ_c corresponds to the configuration with N director half-turns in the layer when it is at an instability point. We also record here that for the typical value $K_{22}=5\times10^{-7}$ dyn and anchoring strength

$W=10^{-3}$ erg cm^{-2}, the criterion $S_d > 1/2\pi$ is certainly satisfied for cell thicknesses $d < 30$ μm.

The pitch in the layer just before the jump, p_d, and the corresponding natural pitch, p, are expressed by φ_c via the following formulas:

$$p_d(T_c) = 2d/(N + \varphi_c/\pi),$$

$$p(T_c) = 2d/[N + \varphi_0(T_c)/\pi], \tag{2.8}$$

where T_c is the jump temperature. The angle $\varphi_0(T_c)$ (the natural one at the jump point temperature) is given by the formula

$$\varphi_0(T_c) = \varphi_c + \frac{d}{K_{22}}\left[\frac{\partial W_s(\varphi)}{\partial \varphi}\right]_{\varphi=\varphi_c}. \tag{2.9}$$

The value of φ after the jump, denoted by φ_j, which is basically connected to the pitch $p_{dj}(T_c)$ in the layer after the jump, is determined by the solution of the equation

$$\frac{\partial W_s(\varphi)}{\partial \varphi} + \frac{K_{22}}{d}[\varphi - \varphi_0(T_c) + \pi] = 0, \tag{2.10}$$

where $\varphi_0(T_c)$ is determined by Eq. (2.9). The critical angle φ_c for identical anchoring at both surfaces is determined only by the shape of the anchoring potential [1,2]; nevertheless, for different anchoring at the surfaces it may be dependent on other parameters of the problem.

Substitution of Eq. (2.6) into Eq. (2.1) yields the following equation for the dynamical variable $\varphi(t)$:

$$\frac{d\varphi}{dt} = -\frac{3}{d\gamma_1}\frac{dF}{d\varphi}. \tag{2.11}$$

As mentioned in the Introduction, we shall select a family of simple model potentials which depend on the parameter W (and, in Sec. III below, on n), which characterizes the depth of the surface potential wells. In such instances, $W_s(\varphi) = Wf(\varphi)$, where $f(\varphi)$ is some dimensionless function of φ. For example, for the Rapini-Papoular potential [3,19], $f(\varphi) = -(1/2)\cos^2(\varphi)$. In this case, Eq. (2.11) can be rewritten in the simple dimensionless form

$$\frac{d\varphi}{d\hat{t}} = -3\frac{S_d}{d\gamma_1}\frac{d\hat{F}}{d\varphi}, \tag{2.12}$$

where the dimensionless free energy $\hat{F}$ is

$$\hat{F} = f(\varphi) + \frac{S_d}{2}[\varphi - \varphi_0(T)]^2, \tag{2.13}$$

and the dimensionless time $\hat{t}$ and the dimensionless parameter S_d are defined by, respectively,

$$\hat{t} = t\frac{\pi^2 K_{22}}{\gamma_1 d^2}, \quad S_d = \frac{K_{22}}{Wd}. \tag{2.14}$$

Equation (2.12) allows us to determine the solution φ implicitly, by integration over φ from $t=0$ when the director angle φ at the surface is equal to the critical value φ_c, up to $\varphi(\hat{t})$ corresponding to the time $\hat{t}$. As discussed in detail in our previous article [7], the critical angle φ_c corresponds to the instability point where the solution for φ, which minimizes the free energy (2.5) [or Eq. (2.13)], loses its stability.

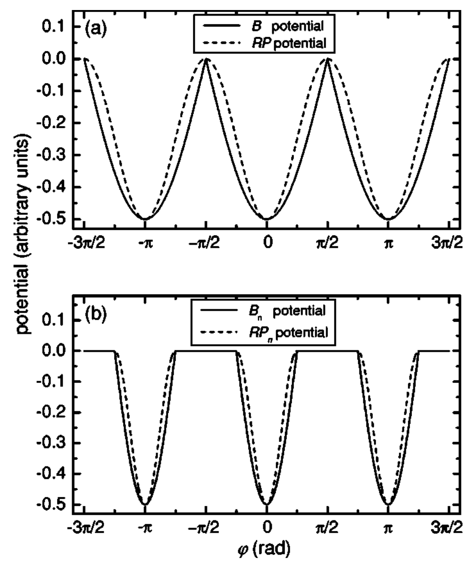

FIG. 2. Qualitative plots of (a) the Rapini-Papoular (RP) and the B potentials, and (b) the Rapini-Papoular-like (RP_n) and the B-like (B_n) potentials. These examples are for $n=2$.

In a similar way, one can also define the duration of the jump τ [7] that is given by the same integration of Eq. (2.11) with the upper limit being replaced by φ_j, which is the angle reached at the final equilibrium state after the jump. This results in the duration τ of the jump being given by

$$\tau = -\frac{d\gamma_1}{3}\int_{\varphi_c}^{\varphi_j}\left[\frac{dF}{d\varphi}\right]^{-1} d\varphi. \tag{2.15}$$

One notes (see Sec. III) that the values of φ_c and φ_j depend strongly upon the shape of the anchoring potential, that is, upon the form of the function $f(\varphi)$ given in the present model. In [7], we considered two types of anchoring potential. The first one was the well-known Rapini-Papoular (RP) potential [3,4,19,27] given by [see Fig. 2(a)]

$$W_s(\varphi) = -\frac{W}{2}\cos^2(\varphi), \tag{2.16}$$

which, for small φ, is simply the first term in the systematic Fourier expansion of any surface potential which depends only on the angle φ. The RP potential clearly has period π. The other potential considered was the so-called B potential [7], which can be expressed as [see Fig. 2(a)]

$$W_s(\varphi) = -W\left[\cos^2\left(\frac{\varphi}{2}\right) - \frac{1}{2}\right], \quad -\frac{\pi}{2} < \varphi < \frac{\pi}{2}. \tag{2.17}$$

The B potential also has period π when it is continued periodically for $|\varphi| > \pi/2$ according to the relation $W_s(\varphi) = W_s(\varphi - \pi)$.

The RP and B potentials are very similar in the vicinity of the minimum point $\varphi=0$, but in the region around the edge of the potential well, i.e., near the *maximum* point (at $\varphi = \pi/2$), the behavior of the B potential is very much different from that of the RP potential. The RP potential is very smooth everywhere, and is characterized by the same curvature at the minimum and maximum points. In contrast, the B potential has a discontinuous first derivative at the edge of the potential well, that is, at the maximum point of the potential. Physically, this means that the edge of such a potential is very "sharp" (i.e., the curvature is very large), and the torque acting on the director rapidly changes sign. In this article, we consider a complementary class of "narrow" potentials which are characterized by having a potential well width less than π and a very broad maximum around $\varphi = \pi/2$ (very low curvature) where there is no restoring torque, i.e., where every value of the angle φ in the "broad maximum" region is marginally stable.

III. NARROW WELL MODEL ANCHORING POTENTIALS

The model anchoring potentials to be introduced in this section are natural generalizations of the RP and B potentials studied in [7]. These new generalized model anchoring potentials will be employed to examine the temperature behavior of the cholesteric helix in a planar cholesteric layer of finite thickness having finite strength of anchoring at one of its boundary surfaces and infinite strength at the other (see Fig. 1).

As in [7], we restrict the analysis of the temperature variations of the director configuration in the layer by assuming that the pitch jump mechanism is connected with the director overcoming the anchoring barrier at the surface and, moreover, that liquid-crystal thermal fluctuations may be neglected. Our main attention follows the approach contained in [1,2,7] and will be concentrated on the transitions between N and $N+1$ half-turns of the director in the layer which proceed without strong local disturbances of the director configuration.

As was shown in [1,2], the variations of the pitch in the layer and, in particular, hysteresis, are determined by the dimensionless parameter $S_d = K_{22}/Wd$, where W is the depth of the anchoring potential. These variations are rather universal because they are not directly dependent upon the sample thickness. This means that for any specific form of the anchoring potential in expression (2.5), Eq. (2.7) may be transformed to a form in which the parameters d, K_{22}, and W of the problem occur only in combinations which reduce to the dimensionless parameter S_d.

Note that, in principle, the anchoring potential may be reconstructed from experimental measurements of the temperature dependence of the angle φ by fitting the measured values to the solution of Eq. (2.7), where $W_s(\varphi)$ should be assumed as an unknown function subjected to determination. However, as a first step it is more practical to compare the measured values of φ with the solutions of Eq. (2.7) by means of some trial functions for $W_s(\varphi)$. In other words, we have to adopt some model potentials for the surface anchoring which have shapes that are reasonably acceptable from the physical point of view.

We now apply the general relations from the previous section to specific shapes of the surface anchoring potential introduced in the following two subsections. These are natural generalizations of the B potential (introduced in [7]) and the RP potential.

A. B_n potential

The B_n potential is defined by

$$W_s(\varphi) = -W\left[\cos^2\left(\frac{n\varphi}{2}\right) - \frac{1}{2}\right] \quad \text{if } -\frac{\pi}{2n} < \varphi < \frac{\pi}{2n}, \tag{3.1}$$

$$W_s(\varphi) = 0 \quad \text{if } \frac{\pi}{2n} < |\varphi| < \frac{\pi}{2}, \tag{3.2}$$

and is continued periodically for $|\varphi| > \pi/2$, according to the relations $W_s(\varphi) = W_s(\varphi - \pi)$, where $n > 1$, as shown in Fig. 2(b). The case for $n=1$ corresponds to the B potential shown in Fig. 2(a). The critical angle for the B_n potential is given by $\varphi_c = \pi/2n$. Depending on the value of the parameter S_d, there are two possible cases for the value of the post-jump angle φ_j. If $S_d < n^2/[2\pi(n-1)]$, then the deviation angle of the director remains inside the anchoring potential well after the jump and φ_j is determined from Eq. (2.7) with $\varphi_0(T)$ replaced by $\varphi_0(T_c) - \pi$. If $S_d > n^2/[2\pi(n-1)]$, then the deviation angle of the director occurs outside of the anchoring potential well after the jump and φ_j is determined by the free rotation angle $\varphi_0(T_c)$ at the point (temperature) of the jump, i.e., by the angle of the director orientation at the surface in the absence of anchoring, which is given by expression

$$\varphi_0(T_c) = \frac{\pi}{2n} + \frac{n}{2S_d}. \tag{3.3}$$

This means that if $S_d > n^2/[2\pi(n-1)]$, then the jump ends at $\varphi_j = \varphi_0(T_c) = \pi/2n + n/2S_d$.

The free energy (2.5) for the B_n potential accepts the form

$$\frac{F(T)}{W} = -\left[\cos^2\left(\frac{n\varphi}{2}\right) - \frac{1}{2}\right] + \frac{S_d}{2}[\varphi - \varphi_0(T)]^2 \quad \text{if } -\frac{\pi}{2n} < \varphi < \frac{\pi}{2n}, \tag{3.4}$$

$$\frac{F(T)}{W} = \frac{S_d}{2}[\varphi - \varphi_0(T)]^2 \quad \text{if } \frac{\pi}{2n} < |\varphi| < \pi - \frac{\pi}{2n}. \tag{3.5}$$

Naturally, the surface term in the free energy (3.4) [i.e., the first term on the right-hand side of Eq. (3.4)] continued as a function of φ outside its given limitations has to satisfy the periodic conditions formulated above on the anchoring potential. Equation (2.7) determines the equilibrium value of φ in the layer as a function of the temperature [or $\varphi_0(T)$] and is of the form

$$\sin(n\varphi) + \frac{2S_d}{n}[\varphi - \varphi_0(T)] = 0 \quad \text{if } -\frac{\pi}{2n} < \varphi < \frac{\pi}{2n}, \tag{3.6}$$

$$\varphi = \varphi_0(T) \quad \text{if } \frac{\pi}{2n} < |\varphi| < \pi - \frac{\pi}{2n}. \tag{3.7}$$

If $S_d < n^2/[2\pi(n-1)]$, then the deviation angle of the director remains inside the anchoring potential well after the jump, and so φ is restricted by the condition $-\pi/2n < \varphi < \pi/2n$. For this case, the deviation angle φ_j after the jump is determined from the expression

$$\sin(n\varphi_j) + \frac{2S_d}{n}[\varphi_j - \varphi_0(T_c) + \pi] = 0, \tag{3.8}$$

where φ_j is inside the potential well, i.e., $-\pi/2n < \varphi_j < \pi/2n$. It should be remembered here again that the angles φ, φ_j, and $\varphi_0(T)$ are measured relative to the alignment direction (cf. Fig. 1), so that after the jump they are reduced by π. For the case $S_d > n^2/[2\pi(n-1)]$, as has been mentioned previously, $\varphi_j = \varphi_0(T_c) = \pi/2n + n/2S_d$.

B. RP$_n$ potential

In a similar fashion, we may define the RP$_n$ potential by

$$W_s(\varphi) = -\frac{W}{2}\cos^2(n\varphi) \quad \text{if } -\frac{\pi}{2n} < \varphi < \frac{\pi}{2n}, \tag{3.9}$$

$$W_s(\varphi) = 0 \quad \text{if } \frac{\pi}{2n} < |\varphi| < \pi - \frac{\pi}{2n}, \tag{3.10}$$

which is continued periodically for $|\varphi| > \pi/2$ according to the relation $W_s(\varphi) = W_s(\varphi - \pi)$, where $n > 1$, as shown in Fig. 2(b). The case for $n=1$ corresponds to the RP potential shown in Fig. 2(a).

For RP$_n$ potentials, a jump-wise transition occurs if $S_d < n^2$. In such cases, the critical angle is dependent on S_d and is determined from the expression

$$\varphi_c = \frac{1}{2n}\left[\arccos\left(-\frac{S_d}{n^2}\right)\right]. \tag{3.11}$$

There are two possible cases depending on the value of the parameter S_d. If $S_d < (1/2)[n\sin(2n\varphi_c)]/\{\pi[1-1/(2n)]-\varphi_c\}$, then the deviation angle of the director remains inside the anchoring potential well after the jump and φ_j is determined from equations analogous to Eqs. (2.7) and (3.8). If $n^2 > S_d > (1/2)[n\sin(2n\varphi_c)]/\{\pi[1-1/(2n)]-\varphi_c\}$, then the deviation angle of the director occurs outside the anchoring potential well after the jump and φ_j is determined by $\varphi_0(T_c)$, i.e., by the angle of the director orientation at the surface in the absence of anchoring, which is given by the expression

$$\varphi_0(T_c) = \varphi_c + \frac{n}{2S_d}\sin(2n\varphi_c). \tag{3.12}$$

In the case when $S_d > n^2$, there are no jumps for the RP$_n$ potentials and the variations of φ with temperature are smooth. This is an observable consequence of the difference between the RP$_n$ and B_n potentials because jumps exist for B_n potentials at any value of the parameter S_d. The free energy (2.5) for the RP$_n$ potential accepts the form

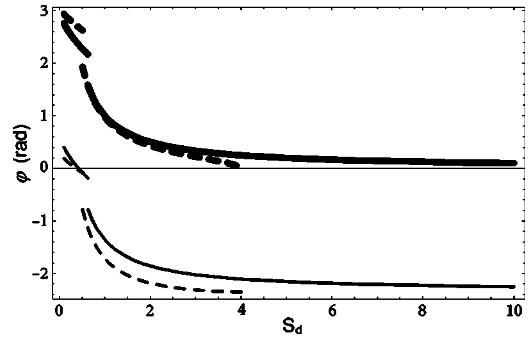

FIG. 3. Post-jump angle φ_j (thin lines) and the angular width of the jump $\varphi_j - \varphi_c$ (bold lines) as functions of S_d, calculated for the RP$_n$ (dashed lines) and B_n potentials (solid lines) when $n=2$.

$$\frac{F(T)}{W} = -\frac{1}{2}\cos^2(n\varphi) + \frac{S_d}{2}[\varphi - \varphi_0(T)]^2 \quad \text{if } -\frac{\pi}{2n} < \varphi < \frac{\pi}{2n}, \tag{3.13}$$

$$\frac{F(T)}{W} = \frac{S_d}{2}[\varphi - \varphi_0(T)]^2 \quad \text{if } \frac{\pi}{2n} < |\varphi| < \pi - \frac{\pi}{2n}. \tag{3.14}$$

The surface term in the free energy described by Eqs. (3.13) and (3.14) [i.e., the first term on the right-hand side of [Eq. (3.13)] continued as a function of φ outside its limitations has to satisfy the periodic conditions formulated above for the anchoring potential. As before, Eq. (2.7) determines the equilibrium value of φ in the layer as a function of the temperature [or $\varphi_0(T)$] and has, in this situation, the form

$$\sin(2n\varphi) + \frac{2S_d}{n}[\varphi - \varphi_0(T)] = 0 \quad \text{if } -\frac{\pi}{2n} < \varphi < \frac{\pi}{2n}, \tag{3.15}$$

$$\varphi = \varphi_0(T) \quad \text{if } \frac{\pi}{2n} < |\varphi| < \pi - \frac{\pi}{2n}. \tag{3.16}$$

If $S_d < (1/2)[n\sin(2n\varphi_c)]/\{\pi[1-1/(2n)]-\varphi_c\}$, then φ remains inside the anchoring potential well after the jump, and

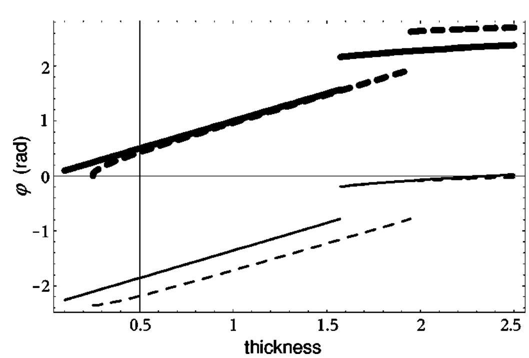

FIG. 4. Post-jump angle φ_j (thin lines) and the angular width of the jump $\varphi_j - \varphi_c$ (bold lines) as functions of the layer thickness (normalized by the penetration length K_{22}/W) calculated for the RP$_n$ (dashed lines) and B_n potentials (solid lines) when $n=2$.

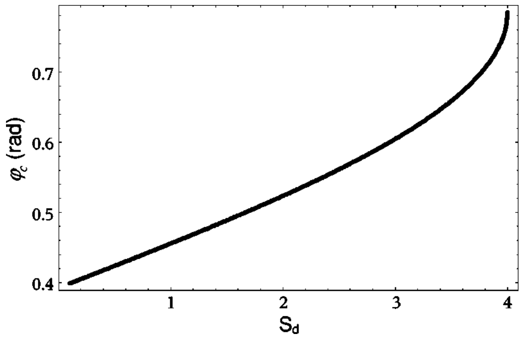

FIG. 5. The dependence of the critical angle φ_c upon S_d for the RP_n potential when $n=2$.

is therefore restricted by the condition $-\pi/2n < \varphi < \pi/2n$. For this case, the deviation angle φ_j after the jump is determined from the expression

$$\sin(2n\varphi_j) + \frac{2S_d}{n}[\varphi_j - \varphi_0(T_c) + \pi] = 0, \qquad (3.17)$$

where φ_j is inside the potential well, i.e., $-\pi/2n < \varphi_j < \pi/2n$, and, more precisely, $-\varphi_c < \varphi_j < \varphi_c$.

The essential difference between the B_n potentials (3.1) and (3.2) and the RP_n potentials (3.9) and (3.10) is in the shape of the wells close to the well edges, just as in the B and RP potentials, particularly in the values of the anchoring force at the points $|\varphi| = \pi/2n$ beyond which the anchoring is absent over some angular interval. For the B_n potentials, this force reaches a maximum value and is discontinuous there, while for the RP_n potentials the force at such values is zero and continuous.

The expressions in this section may be used for obtaining the dynamical characteristics of the pitch jumps for B_n and RP_n potentials which, in particular, are determined by the initial equilibrium φ_c and final φ_j values of the angle φ in the course of a jump. As examples, φ_j and the angular width of the jump (i.e., $\varphi_j - \varphi_c$) have been calculated as functions of S_d in Fig. 3 and as functions of the layer thickness d in Fig. 4 for the B_n and RP_n potentials when $n=2$. Figure 5 demonstrates the dependence of the critical angle φ_c upon S_d for the RP_n potential when $n=2$; the dependence of φ_c upon the layer thickness is presented in Fig. 6 for this same potential.

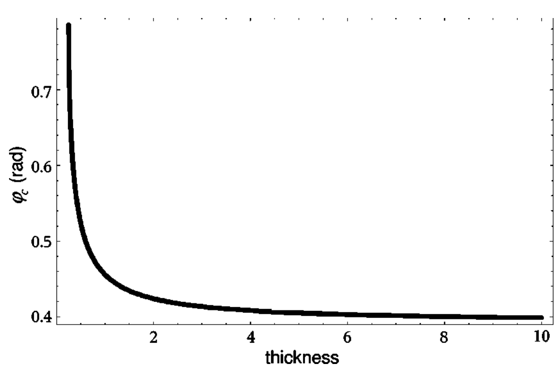

FIG. 6. The dependence of the critical angle φ_c upon the layer thickness (normalized by the penetration length K_{22}/W) for the RP_n potential when $n=2$.

IV. JUMP DYNAMICS FOR NARROW POTENTIALS

We shall describe the jump dynamics of the layer in the course of the transition from the N configuration to the $N+1$ configuration for the narrow B_n potentials (3.1) and (3.2) and the RP_n potentials (3.9) and (3.10), using the same approximation as in [7], that is, we assume that the angular distribution of the director inside the layer is quasistatic and that any hydrodynamic flow is negligible. In our problem the anchoring forces, being localized at the layer surfaces, are nevertheless extremely important because they influence the director distribution in the bulk of the layer. However, as mentioned above, due to the imposed quasistatic approximation of the director inside the layer, the problem may be reduced to that of temporal motion of the director at the layer surface.

We now employ the above general dynamical formulas in the calculation of the temporal characteristics of the jump for the model B_n and RP_n surface anchoring potentials.

A. B_n potential

For the B_n potentials, a jump-wise transition occurs at any value of S_d and the critical angle is given by $\varphi_c = \pi/2n$. If $S_d > n^2/[2\pi(n-1)]$, then at the beginning of the jump $\varphi = \pi/2n$ and the director escapes off the well and the jump ends at $\varphi_j = \varphi_0(T_c) = \pi/2n + n/2S_d$. In this case, $\pi/2n < \varphi < \pi - \pi/2n$ and the temporal dependence of φ is represented by

$$\varphi = [\varphi_0(T_c) - \varphi_c]\left[\frac{\varphi_c}{\varphi_0(T_c) - \varphi_c} + 1 - \exp\left(-\frac{t}{\tau_d}\right)\right],$$

where

$$\tau_d = \frac{d^2\gamma_1}{3K_{22}}. \qquad (4.1)$$

If $S_d < n^2/[2\pi(n-1)]$, then the deviation angle of the director occurs in the next potential well [see Fig. 2(b)] at the end of the jump. When the director is off the well ($\pi/2n < \varphi < \pi - \pi/2n$), the temporal dependence of φ is again represented by the expression (4.1) However, for the director motion inside the next well ($\pi - \pi/2n < \varphi < \pi + \pi/2n$, or what corresponds to $-\pi/2n < \varphi < \pi/2n$ in the $N+1$ configuration of the director), the form of the temporal dependence changes and is given by

$$t = t_f - 2\tau_d S_d \int [n \sin(n\varphi) - n + 2S_d(\varphi + \pi - \pi/2n)]^{-1} d\varphi, \qquad (4.2)$$

where t_f corresponds to the moment when φ reaches the value $\pi - \pi/2n$ [i.e., t_f is determined by Eq. (4.1) when $\varphi = \pi - \pi/2n$] and the integration in Eq. (4.2) runs (in the director configuration with $N+1$ half-turns) from $-\pi/2n$ to φ under the restriction $-\pi/2n < \varphi < \varphi_j < \pi/2n$, where φ_j is determined by the solution of the equation

$$n\sin(n\varphi_j)+2S_d[\varphi_j+\pi-n/(2S_d)-\pi/2n]=0. \quad (4.3)$$

It follows from Eq. (4.1)–(4.3) that the dynamics of pitch jumps in anchoring potentials possessing a narrow angular width of the well differs essentially from the corresponding dynamics for both the RP and B potentials [7].

B. RP_n potentials

For the RP_n potentials, a jump-wise transition occurs if $S_d < n^2$ and the critical angle, which is dependent on S_d, is determined by the expression (3.11). There are two possible cases depending on the value of the parameter S_d. If $S_d < (1/2)[n\sin(2n\varphi_c)]/\{\pi[1-1/(2n)]-\varphi_c\}$, then the deviation angle of the director remains inside the anchoring potential well at the end of the jump and φ_j is determined from Eq. (2.7) with $\varphi_0(T)$ replaced by $\varphi_0(T_c)-\pi$. If $n^2 > S_d > (1/2)\times[n\sin(2n\varphi_c)]/\{\pi[1-1/(2n)]-\varphi_c\}$, then the deviation angle of the director occurs outside the anchoring potential well after the jump and φ_j is determined by $\varphi_0(T_c)$, i.e., by the angle of director orientation at the surface in the absence of anchoring, which is given by the expression (3.12). In the case $n^2 > S_d > (1/2)[n\sin(2n\varphi_c)]/\{\pi[1-1/(2n)]-\varphi_c\}$, with $\varphi_c < \varphi < \varphi_0(T_c) < \pi - \pi/2n$, the time and director deviation angle are interconnected by the expressions

$$t=-2\tau_d S_d\int[n\sin(2n\varphi)-n\sin(2n\varphi_c)+2S_d(\varphi-\varphi_c)]^{-1}d\varphi \quad \text{if } \varphi_c<\varphi<\frac{\pi}{2n}, \quad (4.4)$$

$$\varphi=\varphi_0(T_c)\left[1-\exp\left(-\frac{(t-t_e)}{\tau_d}\right)\right]+\frac{\pi}{2n}\exp\left(-\frac{(t-t_e)}{\tau_d}\right) \quad \text{if } \frac{\pi}{2n}<\varphi<\varphi_0(T_c)<\pi-\frac{\pi}{2n}, \quad (4.5)$$

where, in Eq. (4.4), the lower integration limit is φ_c, and t_e in Eq. (4.5) is determined by Eq. (4.4) when the upper integration limit is set equal to $\pi/2n$.

In the case $S_d < (1/2)[n\sin(2n\varphi_c)]/\{\pi[1-1/(2n)]-\varphi_c\}$, with $\varphi_0(T_c) > \pi-\pi/2n$, the time and director deviation angle are again interconnected by the same expressions in Eq. (4.4) and (4.5) while $\varphi < \pi-\pi/2n$. However, when $\varphi > \pi-\pi/2n$, the time and director deviation angle are provided by the expression

$$t=t_i-2\tau_d S_d\int[n\sin(2n\varphi)-n\sin(2n\varphi_c)+2S_d(\varphi+\pi-\varphi_c)]^{-1}d\varphi \quad \text{if } -\varphi_c<\varphi<\varphi_j, \quad (4.6)$$

where t_i is determined from Eq. (4.5) when $\varphi=\pi-\pi/2n$ and the integration in Eq. (4.6) runs (in the director configuration with $N+1$ half-turns) from $-\pi/2n$ to φ, which is restricted by the condition $-\pi/2n < \varphi < \varphi_j$, where φ_j is determined by the solution of the equation

$$n\sin(2n\varphi)-n\sin(2n\varphi_c)+2S_d(\varphi_j+\pi-\varphi_c)=0, \quad (4.7)$$

with φ_c being determined from Eq. (3.11).

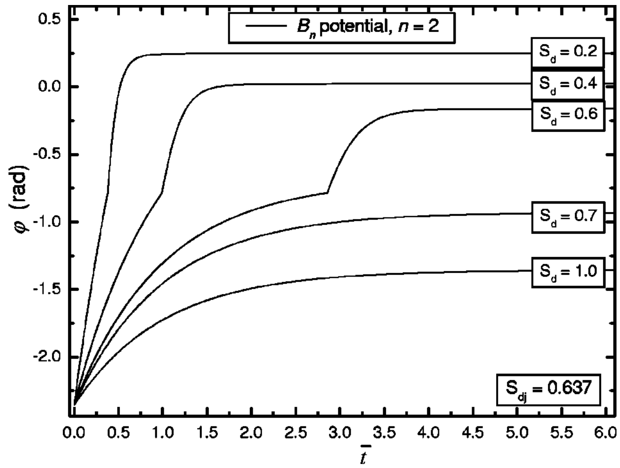

FIG. 7. Temporal behavior (in units of the characteristic time $\tau_d = d^2\gamma_1/3K_{22}$) of the director orientation angle φ at the surface during a jump for the B_n potential for $n=2$ at various values of S_d. There is a critical value $S_{dj}=n^2/[2\pi(n-1)]$ where the nature of the solution changes, as discussed in the text.

The temporal behavior of the director orientation angle φ at the surface during a jump at various values of S_d is presented in Fig. 7 for the B_n potential and in Fig. 8 for the RP_n potential. The calculations were performed according the formulas (4.1)–(4.7) with $\bar{t}=t/\tau_d$, where τ_d is the characteristic time defined in Eq. (4.1)

V. CALCULATIONS AND COMMENTS

The calculations we have performed reveal the observable differences between the RP_n and B_n potentials and also allow a comparison with the results for the RP and B potentials. Before discussing the matter of our main interest, namely, the dynamical properties of the jump, we look at the equilibrium characteristics of the layer for the RP_n and B_n potentials. Figure 3 shows that jump-wise changes of the pitch only exist for the RP_n potential over a limited range of the S_d, unlike that for the B_n potential, for which there is a jump at any value of S_d. For both types of potentials there are jump-wise changes of the post jump angle φ_j which correspond to the minimum value of S_d at which the deviation angle of the director escapes from the potential well after the jump. However, this value of the parameter $S_d=S_{dj}$ at the jump point of φ_j (see Fig. 3) differs: S_{dj} equals 0.637 for the B_n potential and 0.513 for the RP_n potential when $n=2$. This difference is demonstrated in Fig. 3 and also by Fig. 4, where the dependence of the jump angle upon the layer thickness is presented and the difference in the location of the jump point is shown more clearly. This difference reveals itself, sometimes in quite a pronounced way, in the jump dynamics. In general, with growth of n the difference between the B_n and RP_n potentials becomes less pronounced. In particular, the difference in the values of the parameter S_{dj} at the jump point of φ_j decreases. For example, the values of S_{dj} at the jump point of φ_j are 0.712 for the B_n potential and 0.638 for the RP_n potential when $n=3$, and 0.752 and 0.784, respectively, for $n=4$.

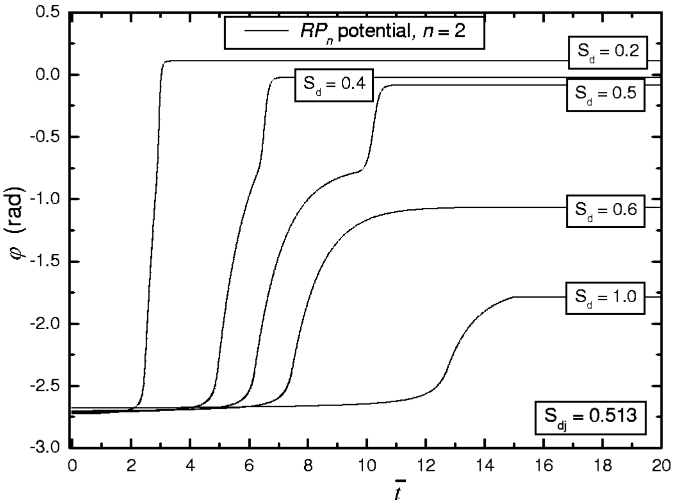

FIG. 8. Temporal behavior (in units of the characteristic time $\tau_d = d^2\gamma_1/3K_{22}$) of the director orientation angle φ at the surface during a jump for the RP_n potential for $n=2$ at various values of S_d. There is a critical value S_{dj} for the jump point φ_j where the nature of the solution changes, as discussed in the text.

Comparison of Fig. 7 and Fig. 8 shows that the jump dynamics is generally slower for the RP_n potential than for the B_n potential. It is especially pronounced for large S_d (weak anchoring). For strong anchoring (corresponding to small values of S_d), the differences becomes less but nevertheless retain features that are quite observable.

Figure 9 presents a comparison of the jump dynamics for the B_n and RP_n potentials on a real time scale for typical values of the experimental parameters involved in possible measurements. Here, the differences in the dynamics for weak anchoring are especially clear. In particular, Fig. 9 indicates that for an experimental distinction to be made between the B_n and RP_n potentials, one should perform an experiment for a weak anchoring, or for a thin sample. (Of course, it is better if both these conditions are fulfilled simultaneously.) The normalized energy $\overline{W} = W/W_0$ has been introduced in Fig. 9, where we have chosen to set $W_0 = 10^{-5}$ J m^{-2}, motivated by typical values for the weak anchoring strength derived from experimental results [10].

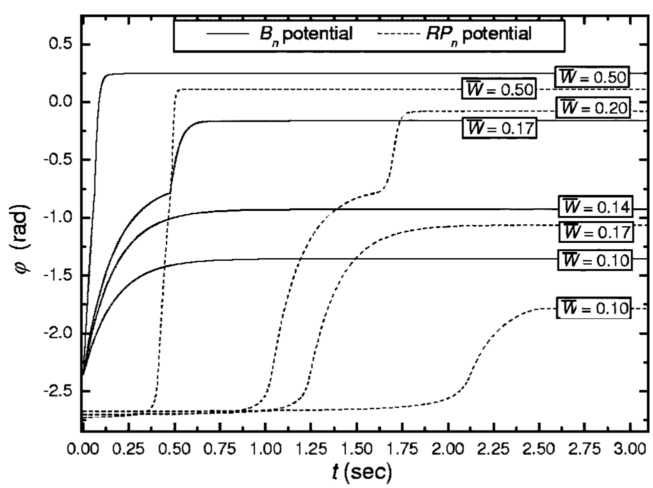

FIG. 9. Comparison of the temporal behavior of the director orientation angle φ at the surface during a jump for the B_n potential and RP_n potential for various values of the normalized anchoring strength $\overline{W}$ and for typical values of the other parameters being set at $d=10$ μm, $\gamma_1=0.05$ Pa s, and $K_{22}=10^{-11}$ N. The normalized anchoring strength is defined by $\overline{W}=W/W_0$, where we have chosen $W_0=10^{-5}$ J m^{-2}.

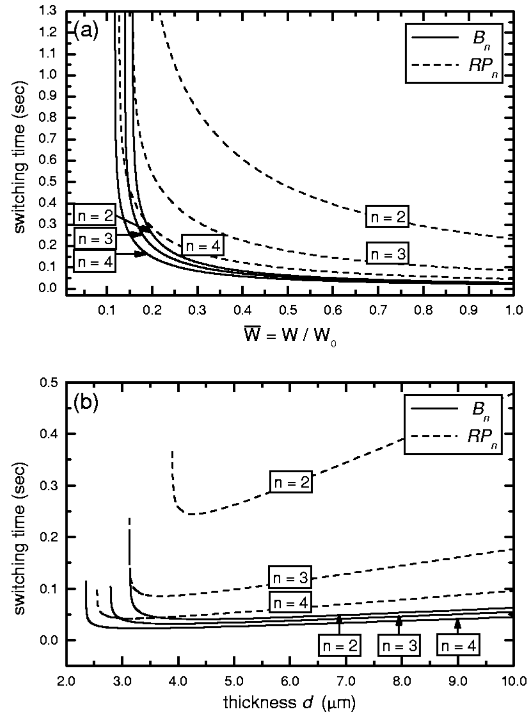

FIG. 10. (a) The dependence of the switching time t_s upon the normalized anchoring strength $\overline{W}$ calculated for the B_n and RP_n potentials when typical values of the other relevant parameters are set to $d=10$ μm, $\gamma_1=0.05$ Pa s, and $K_{22}=10^{-11}$ N. The normalized anchoring strength has been defined by $\overline{W}=W/W_0$, where $W_0=10^{-5}$ J m^{-2}. (b) The dependence of t_s upon the sample thickness d when $W=5\times10^{-6}$ J m^{-2}, with γ_1 and K_{22} as in (a).

Figures 7–9 show that for $S_d < S_{dj}$, the switching time t_s in the jump may be introduced in a natural way as the corresponding time it takes for the deviation angle of the director, after the jump, to reach the orientation that coincides with the edge of the surface anchoring potential well.

For the B_n potential, the corresponding switching time t_s is determined from the expression

$$t_s = -\tau_d \ln[1 + 2\pi S_d(1-n)/n^2]. \tag{5.1}$$

For the RP_n potential, the switching time t_s is determined from the expressions (4.4) and (4.5) if one puts $\varphi=\pi[1-1/(2n)]$ into Eq. (4.5). The dependence of the switching time t_s upon the normalized energy $\overline{W}=W/W_0$, with $W_0=10^{-5}$ J m^{-2}, for the B_n and RP_n potentials (for $n=2$, 3, and 4), is presented in Fig. 10(a) for typical values of the material parameters, as indicated in the caption. Figure 10(b) shows the dependence of t_s upon the sample thickness d when the anchoring strength has been supposed fixed at $W=5\times10^{-6}$ J m^{-2}, with the other parameters as indicated; for

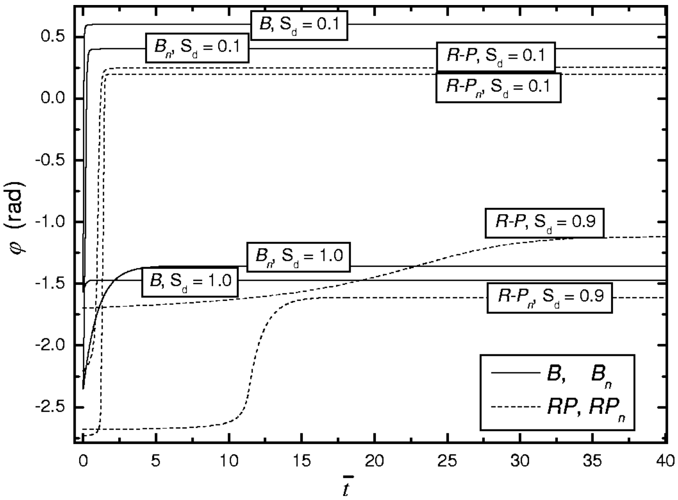

FIG. 11. Comparison of the temporal behavior (in units of the characteristic time $\tau_d = d^2\gamma_1/3K_{22}$) of the director orientation angle φ at the surface during a jump for the B, RP, B_n, and RP$_n$ potentials (for $n=2$) for various values of the parameter S_d. Recall that the anchoring strength increases as S_d decreases.

each value of n there will be a critical thickness of the layer below which the switching time t_s, introduced via Eq. (4.4), (4.5), and (5.1), loses its sense and the temporal characteristic of the jump is then the relaxation time of the process given by τ_d in Eq. (4.1). The critical depth d_j is derived from the critical parameter S_{dj} through the relation $d_j = K_{22}/WS_{dj}$. Note that the switching times at $d=10$ μm displayed in Fig. 10(b) coincide with those displayed in Fig. 10(a) at $\overline{W}=0.5$. As one expects, the switching times are shorter for the B_n potential than for the RP$_n$ potential at the same normalized anchoring strength $\overline{W}$; however, this difference diminishes as S_d decreases, i.e., as the anchoring strength increases. This is evident from the qualitative results presented in Fig. 11, which show the dependence of the deviation angle φ upon the normalized time $\overline{t} = t/\tau_d$, as introduced above. Calculations for Fig. 11 were carried out for the B, RP, B_n, and RP$_n$ potentials (for $n=2$) for the indicated values of S_d.

It should be mentioned here that for the RP$_n$ potential, the integral in Eq. (4.4) diverges at the lower limit φ_c. In fact, with the aid of an elementary expansion and the identity (3.11), it can be shown that the integrand in Eq. (4.4) has a singularity of second order around $\varphi=\varphi_c$. This forces a cutoff at some value of φ which is determined from physical reasons, otherwise the initial motion of the director in the course of a jump occurs infinitely slowly. The evident explanation for this required shift of the lower integration limit to a value slightly above φ_c is the presence of thermal fluctuations in the angle φ, which have so far been neglected in this article. If one estimates these fluctuations in φ according to the phenomenological approach proposed in [2], then one finds that the angular fluctuations in φ due to thermal fluctuations for typical values of the liquid-crystal parameters and for a layer thickness of the order 10 μm exceed 0.01. Therefore, in the calculations of the integral (4.4) it was considered acceptable to introduce a cutoff equal to 0.01, i.e., it was assumed that $\varphi - \varphi_c > 0.01$. A more consistent and rigorous account of thermal fluctuations in the director orientation at the layer surface may be performed in the framework of the approach used in [9].

It is worth noticing that the motion of the director at the beginning of the pitch jump for the RP-like potentials is essentially slower than that for the B-like potentials. This feature may be regarded as a convenient means by which one can experimentally distinguish between these two types of potential.

VI. CONCLUSIONS

The results of this article enable one to shed some light on the form of the anchoring potential for large deviations of the director away from the easy axis (i.e., for that part of the potential which can hardly be tested using conventional equilibrium techniques). For small deviations from the easy axis, all potentials are quadratic in the first approximation. In contrast, the actual form of the potential for large deviations is generally unknown and may differ significantly from the simple RP form. One notes that for certain values of the parameters, the pitch jumps may not occur at all in models that are based on the Rapini-Papoular form of the anchoring potential [1]. It was shown in this paper that these same models are characterized by very large (formally diverging) relaxation times. Both results are in contradiction with existing experimental data, including typical velocities of domain walls. These peculiar features of the model are related to the specific form of the Rapini-Papoular potential at large deviation angles from the equilibrium and deserve a more detailed discussion. In this article, we have undertaken a comparative study of the pitch jumps for four different types of potential: the standard RP potential and the B potential [see Fig. 2(a)], which has a discontinuity of the derivative at the point of a maximum (i.e., where there is very large curvature), and the narrow well RP and B potentials, which have very low curvature around the midpoint between two successive wells [see Fig. 2(b)]. The main qualitative results are summarized in Fig. 11. First, one notes that the dynamics of the director is qualitatively different in the cases of strong and weak anchoring. For strong anchoring (i.e., low values of the parameter S_d), the director dynamics and the corresponding relaxation times are qualitatively the same for all types of potentials, and although there are quantitative differences, one can hardly extract any qualitative information about the actual form of the anchoring potential from the experimental data. In contrast, for weak anchoring the dynamics of the director strongly depends on the type of the potential. For the RP potential, the relaxation time is very long and should vary significantly from one measurement to another because, as discussed in Sec. V, the director relaxation in this case is triggered by a fluctuation of the azimuthal angle ϕ. For narrow RP potentials, the general behavior is similar but the response times are slightly longer than those for the RP potential. These qualitative features can be observed experimentally if RP-type potentials represent good models for actual potentials. In contrast, the response times are significantly shorter for B-type potentials compared to the RP-type potentials and the dynamics is much more regular.

For narrow B_n potentials, the response time is longer compared to the B potential and this again may be a qualitative signature of the corresponding form of the potential.

Despite the fact that the specific calculations of the jump dynamics were performed under simplifying assumptions, there is no doubt that the qualitative features of the jump dynamics discussed above will remain valid for jump-wise phenomena in general. One simplification used was to ignore any possible effects due to backflow. As mentioned at the beginning of Sec. II, backflow in the classical twist geometry for nematics can be safely neglected: the qualitative results are unaffected. Although backflow cannot be ruled out for the problem discussed here, we anticipate that the situation will be similar to that for nematics in a classical twist geometry because the key elastic mechanism in the jump dynamics is related to twist.

The comparison of Figs. 7–11 (see also Figs. 3 and 4) shows that experimental measurements would allow one to obtain a qualitative conclusion about the applicability of the RP potential in the description of jumps (for $S_d > 1$, the jumps are absent for the RP potential) and, moreover, would determine what kind of narrow well RP-like and B-like potentials, as introduced above, may be applicable in the description of jumps. The results presented here show that experimental investigations of the pitch jump dynamics will provide a unique opportunity to study the actual shape of the anchoring potential for large angular deviations of the director away from the alignment direction.

We conclude this section with one final remark. Notice that the calculations were mainly performed over a range of values for the parameter S_d for which the pitch jump with $\Delta N = 1$ proceeds to the equilibrium state of the layer, i.e., for the situation where the free energy corresponding to the final director configuration is less than the free energy of any other possible configuration. The corresponding limitation demands that the parameter S_d exceeds some critical value S_{dc}. For the B-like potentials, $S_{dc} = n/3\pi$. For the RP-like potentials, S_{dc} is slightly less than this because it is determined by the relation $S_{dc} = n \sin[\arccos(-S_{dc}/n^2)]/3\pi$, where the case for $n = 1$ corresponds to that arising from the RP and B potentials.

ACKNOWLEDGMENTS

The authors are grateful for the UK EPSRC Grant No. GR/S34311/01, which enabled this work to be undertaken. The partial funding of this work by the RFBR Grant No. 03-02-16173 is greatly appreciated by V.A.B.

APPENDIX

In this article, we have assumed that the director configuration inside the cell remains quasistatic during the jump, and therefore the azimuthal angle $\phi(z,t) = (z/d)\varphi(t)$. In this simple case, the whole director configuration is described by the single variable $\varphi(t)$. This approximation can be justified by considering a more general case when the director configuration is not quasistatic and is specified by the function $\phi(z,t)$, which satisfies the Lagrange equation (see, for example, Vertogen and de Jeu [28])

$$\frac{1}{2}\frac{\partial D}{\partial \dot{\phi}} = -F_{el,\phi}, \tag{A1}$$

where we have neglected, as usual, the inertial terms. Here $F_{el,\phi}$ is the generalized elastic force density

$$F_{el,\phi} = \frac{\partial f_d}{\partial \phi} - \partial_\alpha \frac{\partial f_d}{\partial(\partial_\alpha \phi)}, \tag{A2}$$

where f_d is the distortion free energy. Equation (2.2) can be rewritten in a simple explicit form if one assumes that the director always remains in the xy plane, i.e., $\mathbf{n} = \hat{\mathbf{x}} \cos \phi + \hat{\mathbf{y}} \sin \phi$. In the absence of any flow, the dissipation function $\mathcal{D}$ is again given by Eq. (2.6), and the distortion free energy f_d is given by

$$f_d = \frac{1}{2}K_{22}\left(\frac{\partial \phi}{\partial z}\right)^2 + q_0 K_{22}\frac{\partial \phi}{\partial z}. \tag{A3}$$

Substituting Eqs. (2.6) and (A3) into Eqs. (A1) and (A2), one obtains the following equation for $\phi(z,t)$ in the bulk:

$$\gamma_1 \frac{\partial \phi}{\partial t} = K_{22}\frac{\partial^2 \phi}{\partial z^2}. \tag{A4}$$

The diffusion equation (A4) describes the relaxation and propagation of the azimuthal angle profile inside the liquid-crystal cell. This equation has the same form as the one used in the theory of the Freedericksz transition [3,27]. One notes that the dynamics of the angle $\phi(z,t)$ is strongly influenced by the boundary conditions. We assume that the anchoring is strong at the surface $z = 0$. Then $\phi(0,t) = 0$ for all t. The surface $z = d$ is characterized by the anchoring energy $U_s(\phi)$. The angle $\phi = \phi(d,t)$ at this surface satisfies the well known boundary condition

$$\frac{dU_s}{d\phi} = K_{22}\left(\frac{\partial \phi}{\partial z} - q_0\right) \tag{A5}$$

at $z = d$. One notes that Eq. (A4) has the stationary solution

$$\phi_{eq}(z) = z\phi_0/d, \tag{A6}$$

which satisfies the boundary condition $\phi(0,t) = 0$ and corresponds to the quasiequilibrium structure. The time-dependent solution of Eq. (A4), which satisfies the boundary condition $\phi(0,t) = 0$, can be written as a sum of the stationary solution (A6) and a sum of eigenmodes with decreasing relaxation times to reveal that

$$\phi(z,t) = z\phi_0/d + \sum_n A_n \exp\left(-\frac{n^2 K_{22} t}{\gamma_1}\right)\sin(nz). \tag{A7}$$

Here the constants ϕ_0 and n should be determined using the second boundary condition derived from Eq. (A5), namely,

$$\frac{dU_s}{d\phi} = K_{22}(\phi_0/d - q_0) + K_{22}\sum_n nA_n \exp\left(-\frac{n^2K_{22}t}{\gamma_1}\right)\cos(nd), \tag{A8}$$

which should be valid for all t.

It should be noted that for sufficiently large values of ϕ, the anchoring potential is always a nonlinear function of ϕ and thus ϕ_0 and n can only be determined numerically. On the other hand, these constants can be estimated for some limiting cases. For example, in the case of strong anchoring at the surface $z=d$ the angle $\phi+\phi(d,t)$ also vanishes and the constants n are expressed as $n=\pi k/d$, where $k=1,2,3,\ldots$. In this case, the largest relaxation time is $\tau_0=\gamma_1 d^2/K_{22}\pi^2$, which is the unit of time used in [3] and is $3/\pi^2$ times the unit of time τ_d used in this article. One can readily see from Fig. 11 that in the case of weak anchoring, the relaxation time obtained in the quasistatic approximation is of the order of $10\tau_0$. This means that for weak anchoring, the relaxation times of some eigenmodes in Eq. (A7) may be much larger than τ_0 and therefore some constants n may be much smaller than π/d. This result enables one to derive the quasistatic approximation from the general equation (A7) and to investigate its region of validity.

Thus let us assume that some relaxation times in Eq. (A7) are much larger than τ_0. This means that some $n \ll \pi/d$. In this case, the general solution can be expressed as a sum of two terms $\phi(z,t)=\phi_1(z,t)+\phi_2(z,t)$, where the function $\phi_2(z,t)$ is a sum of all eigenmodes [from the general equation (A7)] with short relaxation times $\tau<\tau_0$ or $\tau\sim\tau_0$. By contrast, the function $\phi_1(z,t)$ contains only modes with large relaxation times $\tau \gg \tau_0$. At the time scale $t\gg\tau_0$, the function ϕ_2 vanishes and the azimuthal angle profile is approximately given by $\phi(z,t)\approx\phi_1(z,t)$, where $\phi_1(z,t)$ is expressed in the form of Eq. (A7) with all $n\ll\pi/d$. Then the argument of the sine function in Eq. (A7) satisfies $nz\ll\pi$ for all $0<z<d$ and therefore $\sin(nz)$ can be approximated as $\sin(nz)\approx nz$. Consequently, the azimuthal angle profile for $t\gg\tau_0$ can be expressed as

$$\phi(z,t) \approx z\phi_0/d + \sum_n A_n \exp\left(-\frac{n^2K_{22}t}{\gamma_1}\right)nz = \varphi(t)z/d, \tag{A9}$$

where $\varphi(t)$ is the azimuthal angle at the surface $z=d$, given by

$$\varphi(t) = \phi_0 + \sum_n A_n \exp\left(-\frac{n^2K_{22}t}{\gamma_1}\right)nd. \tag{A10}$$

One notes that Eq. (A9) has exactly the same form as Eq. (2.2), which describes the azimuthal angle profile in the quasistatic approximation. Thus the quasistatic approximation has been derived directly from the general equation (A7) assuming that there exist relaxation times larger than τ_0. As discussed above, in this approximation the dynamics of the pitch jump is described by Eq. (2.11), which indeed yields sufficiently larger relaxation times in the case of weak anchoring. This conclusion confirms the assumption made in this article upon the angle $\phi(z,t)$.

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